

March 11, 1886.

Professor STOKES, D.C.L., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

- I. "The Influence of Stress and Strain on the Physical Properties of Matter. Part I. Elasticity—*continued*. The Internal Friction of Metals." By HERBERT TOMLINSON, B.A. Communicated by Professor W. GRYLLS ADAMS, M.A., F.R.S. Received February 18, 1886.
(Abstract.)

An abstract of a paper on this subject has been already published,* but the paper itself was withdrawn for the purpose of revision. The fresh experiments which have been for this purpose instituted during the last year were made with improved apparatus, and the coefficient of viscosity of air redetermined, with a view of enabling the author to make more accurate correction for the effect of the resistance of the air.† These more recent experiments on the loss of energy of a torsionally vibrating wire, besides confirming the results of the older ones, as far as the latter have been published, have furnished, more or less in addition, the following facts relating to the internal molecular friction of metals:—

The proportionate diminution of amplitude is independent of the amplitude, provided the deformations produced do not exceed a certain limit. This limit varies with the nature of the metal, and is for nickel very low.

The logarithmic decrement of amplitude increases with the length of the vibration-period, but in a less proportion than the latter, and in a diminishing ratio. The amount of increase of the logarithmic decrement, attending on a given increase through a given range of the vibration-period, varies with the nature of the metal, and with those metals which possess comparatively small internal friction becomes almost insensible. It follows as a consequence that the

* "Proc. Roy. Soc.," vol. 38, p. 42.

† An abstracted account of this redetermination was read before the Royal Society, January 14, 1886.

internal friction of metals differs from the viscosity of fluids, for in cases of damping by the latter the logarithmic decrement is *inversely* as the length of the vibration-period.

Permanent molecular strain resulting from loading not carried to a sufficient extent to produce sensible permanent extension, diminishes the internal friction, and increases the torsional elasticity.

Considerable permanent longitudinal extension and permanent torsion produce increase of internal friction and diminution of torsional elasticity. The effect of torsion is much greater than that of extension, and the increase of internal friction is much greater than the decrease of torsional elasticity. As a consequence wire-drawing, where we have permanent extension and torsion combined, sometimes increases enormously the internal friction; in fact in the case of six different metals, it was found that by careful annealing the internal friction was decreased from *one-half to one-thirtieth* of the original amount of friction of the metals in the hard-drawn condition. Almost equally remarkable is the effect of rapid fluctuations of temperature, even through ranges of only one or two degrees centigrade, in increasing the internal friction.

The internal friction of a metal wire, whether in the hard-drawn or annealed condition, is temporarily decreased, and the torsional elasticity is temporarily increased by loading not carried beyond a certain limit, beyond this limit both the friction and the elasticity become independent of the load.

The "fatigue of elasticity," discovered by Sir William Thomson in metal wires when vibrating torsionally, is not felt, provided the deformations produced do not exceed a certain limit, depending upon the nature of the metal. The above-mentioned limit is extraordinarily low for nickel, so low, indeed, that it is difficult to avoid "elastic fatigue" with this metal. This last consideration, and others founded on the results of experiments on the effects of stress on the physical properties of nickel, tend to show that the molecules of this metal are comparatively easily rotated about their axes.

The author agrees with Prof. G. Wiedemann, that the loss of energy due to internal friction in a torsionally vibrating wire is mainly due to the to-and-fro rotation of the molecules about their axes; any cause, therefore, which increases the molecular rotatory elasticity diminishes the internal friction, and conversely. The author has, by various means, succeeded in bringing down the internal friction to such an extent that, in the case of one wire, it would have required upwards of 15,000 vibrations to diminish the amplitude to one-half of its initial value, provided the vibrations had been executed *in vacuo*.

The molecules of a metal tend to creep into such positions as will ensure a maximum molecular rotatory elasticity, and they can be

assisted in doing so by agitations effected either by thermal or mechanical agency; hence—

Rest after suspension, aided by oscillations at intervals, diminishes the internal friction of a wire which has been recently suspended, or which after a long suspension has been subjected to considerable molecular agitation by either mechanical or thermal agency.

On the contrary, when a maximum molecular rotatory elasticity has been reached, molecular agitation, if carried beyond a certain limit, diminishes the elasticity; hence the results of “fatigue of elasticity;” and hence—

Mechanical shocks and rapid fluctuations of temperature beyond certain limits may considerably increase the internal friction, and, though to a much less extent, diminish the torsional elasticity.

The logarithmic decrement is independent of both the length and diameter of the wire.

II. “On Systems of Circles and Spheres.” By R. LACHLAN, B.A., Fellow of Trinity College, Cambridge. Communicated by Professor A. CAYLEY, F.R.S. Received February 23, 1886.

(Abstract.)

This memoir is an attempt to develop the ideas contained in two papers to be found in the volume of Clifford's Mathematical Papers (Macmillan, 1882), viz., “On Power Coordinates” (pp. 546—555), and “On the Powers of Spheres” (pp. 332—336); the date of the former is stated to be 1866, and of the latter 1868, but the editor explains (see p. xxii, and note, p. 332) that though these papers probably contain the substance of a paper read to the London Mathematical Society, February 27, 1868, “On Circles and Spheres” (“Proc. L. M. S.,” vol. ii, p. 61), they were probably not written out before 1876. It is possible, therefore, that Clifford may be indebted to Darboux for the conception of the “*power* of two circles,” or spheres, as an extension of Steiner's use of the “*power* of a point with respect to a circle. Darboux was the first to give the definition of the power of two circles, in a paper “Sur les Relations entre les Groupes de Points, de Cercles, et de Sphères” (“Annales de l'École Normale Supérieure,” vol. i, p. 323, 1872), in which some theorems analogous to the fundamental theorem of this memoir are proved.

This memoir is divided into three Parts: Part I consists of the discussion of systems of circles in one plane; Part II of systems of circles on the surface of a sphere; and Part III of systems of spheres.

The power of two circles is defined to be the square of the distance between their centres less the sum of the squares of their radii.